Hydrodynamics in Heavy-Ion Collisions

CRC–TR 211
Strong-interaction matter under extreme conditions

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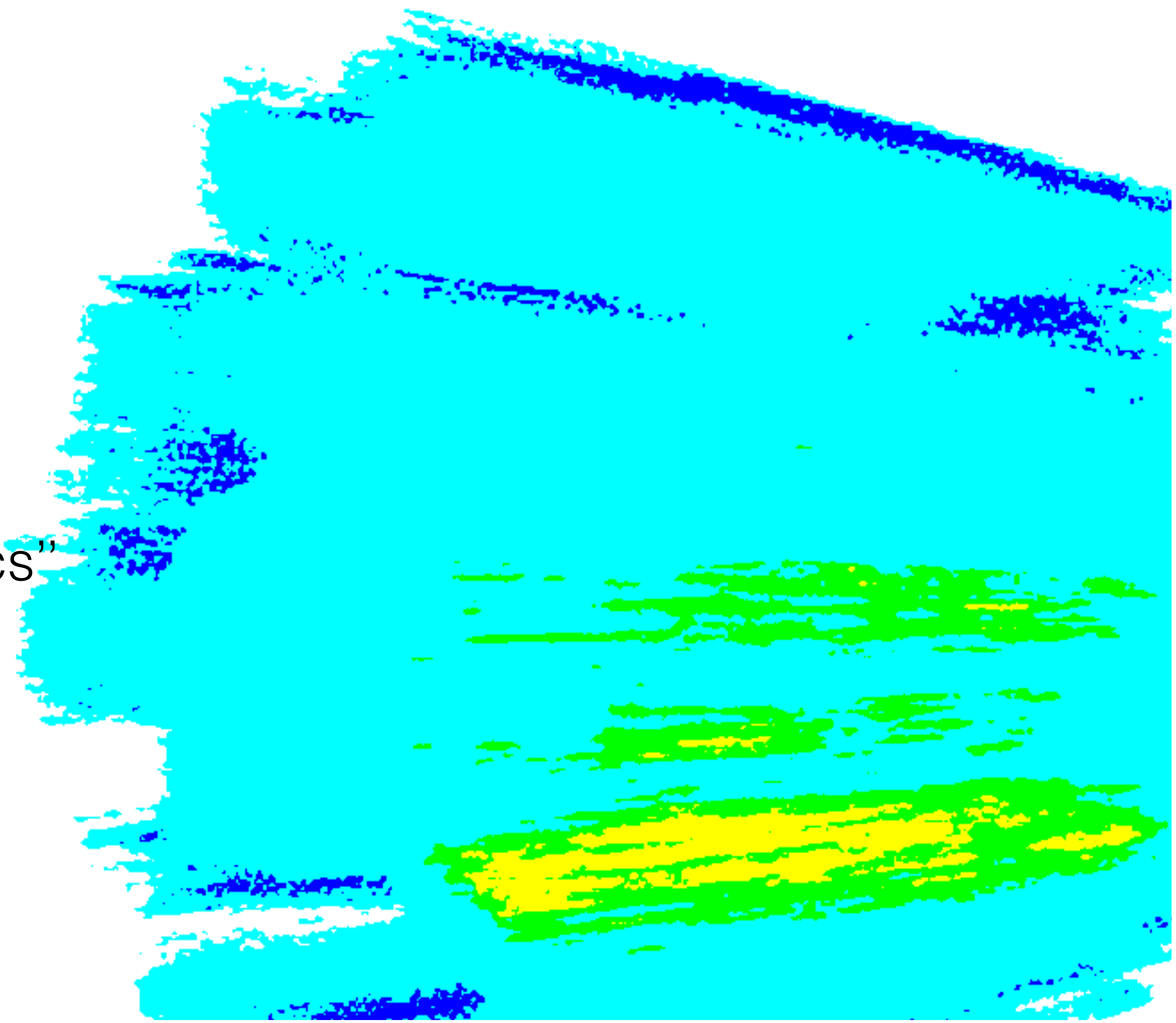

BIP ``Relativistic Hydrodynamics’’
Timisoara May 2026

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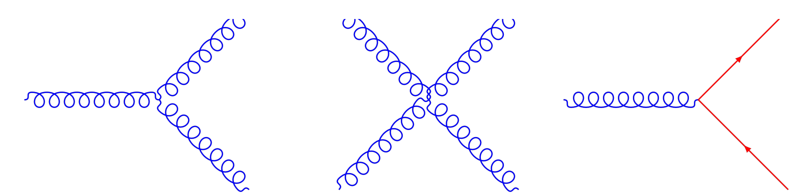
Strong-interaction matter

Strong interactions microscopically described by Quantum Chromo Dynamics (QCD)

$$\mathcal{L}_{QCD} = \sum_{f}^{N_f} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{2} \mathrm{tr} F_{\mu\nu}^2$$

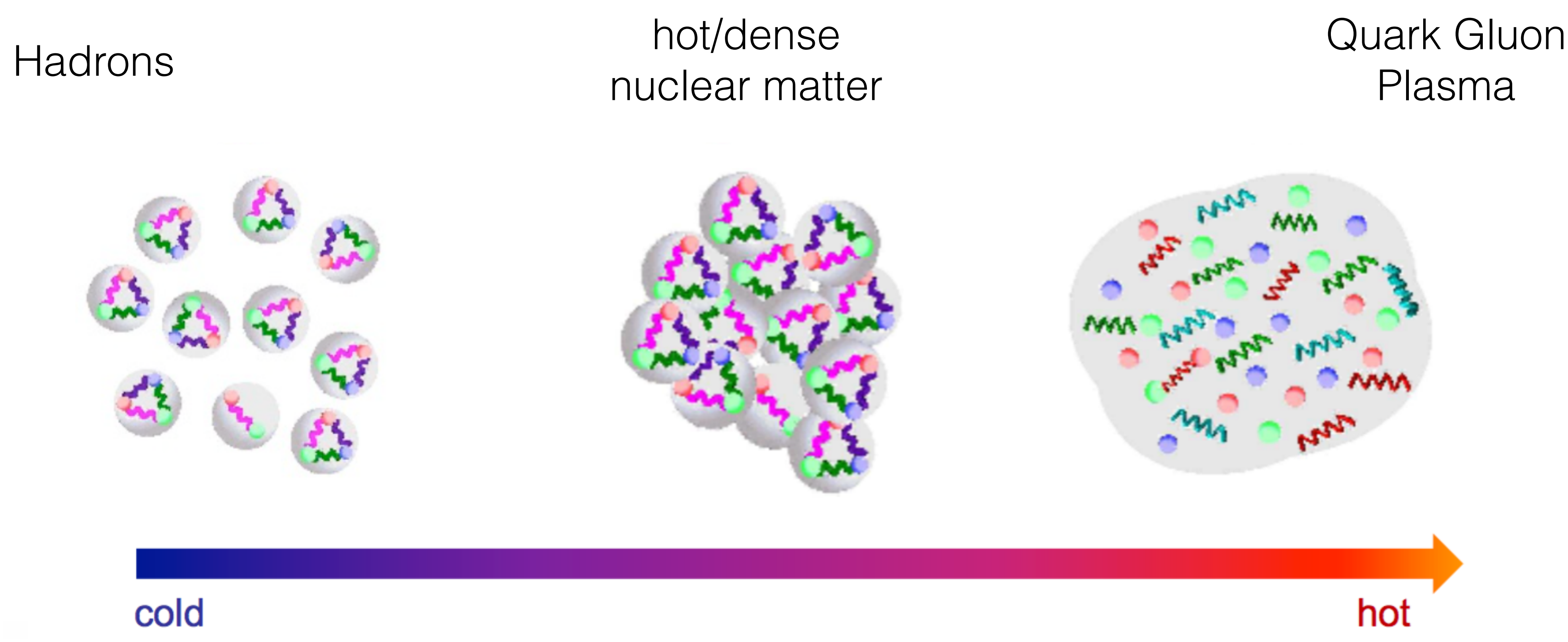


in terms of Quark and Gluon degrees of freedom and their interactions

Non-perturbative nature of QCD gives rise to **confinement** of Quarks & Gluons in hadronic bound states (π,K,p,n,…) in the vacuum

Strong-interaction matter under extreme conditions

Explore dynamics of Quarks & Gluons by heating up/compressing nuclear matter until fundamental constituents are liberated from bound states



Basic idea of HICs: Collide heavy nuclei at high-energies to concentrate large amount of energy in “large” volume (QCD scales) to realize conditions for deconfinement & chiral symmetry restoration

Strong-interaction matter under extreme conditions

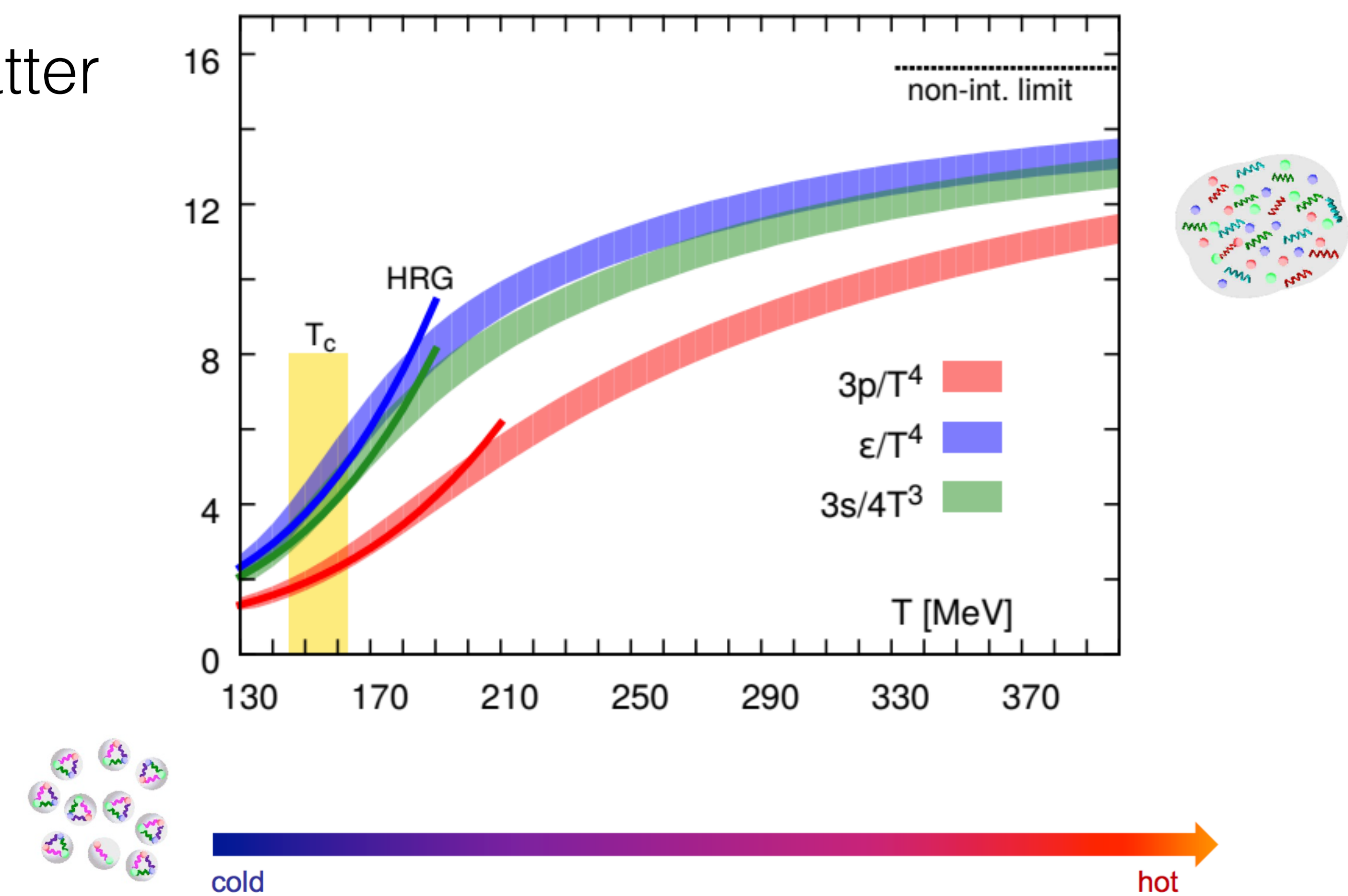
Thermodynamic properties of QCD matter at finite temperature can be calculated from first-principles using lattice QCD (numerical evaluation of QCD partition function)

Energy density, pressure, entropy

$$e(T) = \frac{\pi^2}{30}\nu_{\text{eff}}T^4$$

$$p(T) = e(T)/3\ , s(T) = 4/3e(T)/T$$

for non-interacting bosons

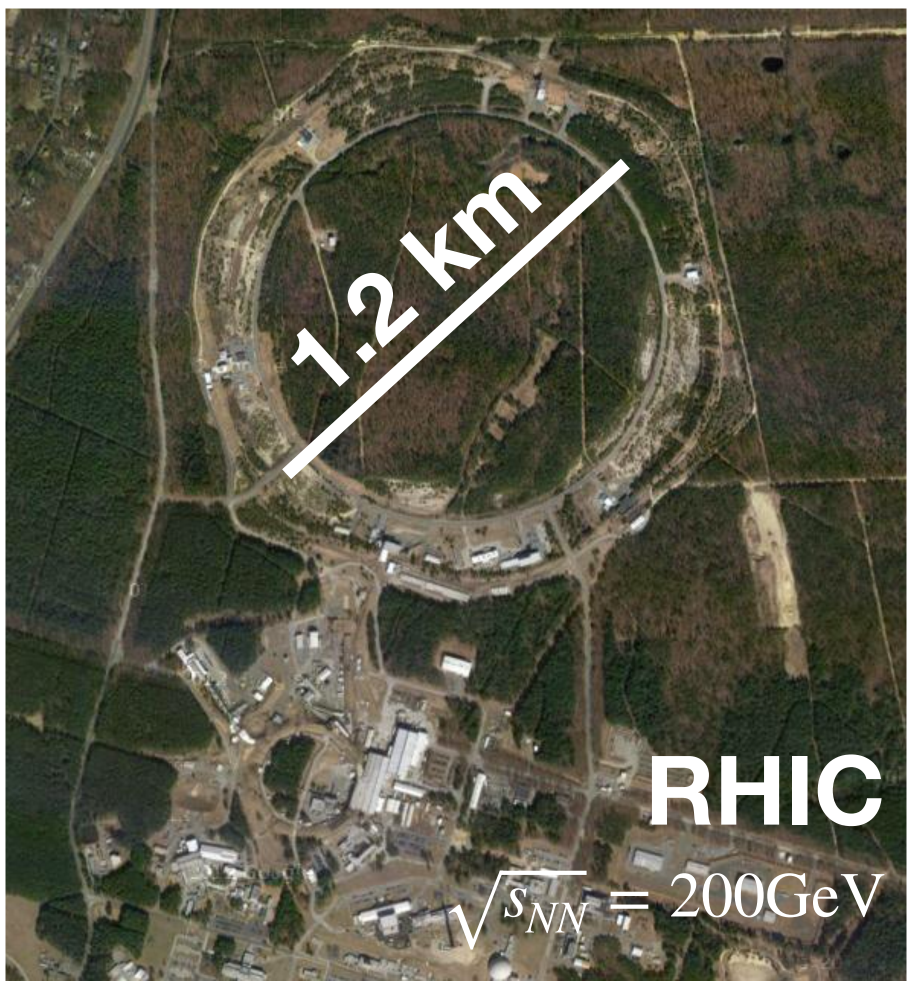


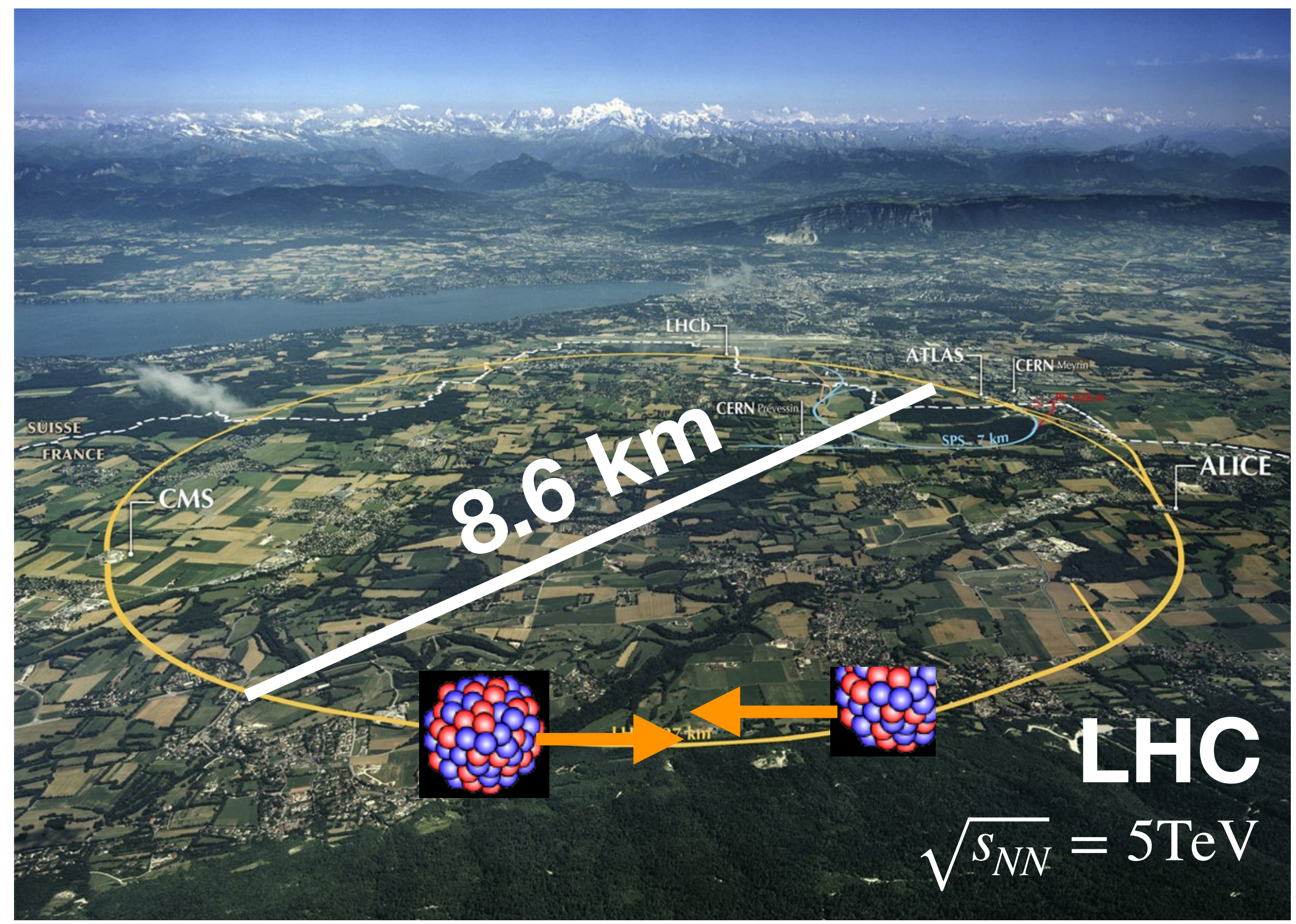
HotQCD Collaboration Phys.Rev. D90 (2014) 094503

Strong rise of eff. number of degrees of freedom suggest transition to Quark-Gluon Plasma (QGP) at temperatures T_C~155 MeV~10^{12} K (more than 10^6 T_{sun})

Heavy-Ion Collisions

Collide heavy (Au,Pb,U,..) and light (Xe,O,p,d,He3,…) nuclei at (ultra)-relativistic energies at RHIC,LHC and SPS, GSI/FAIR, JINR/NICA

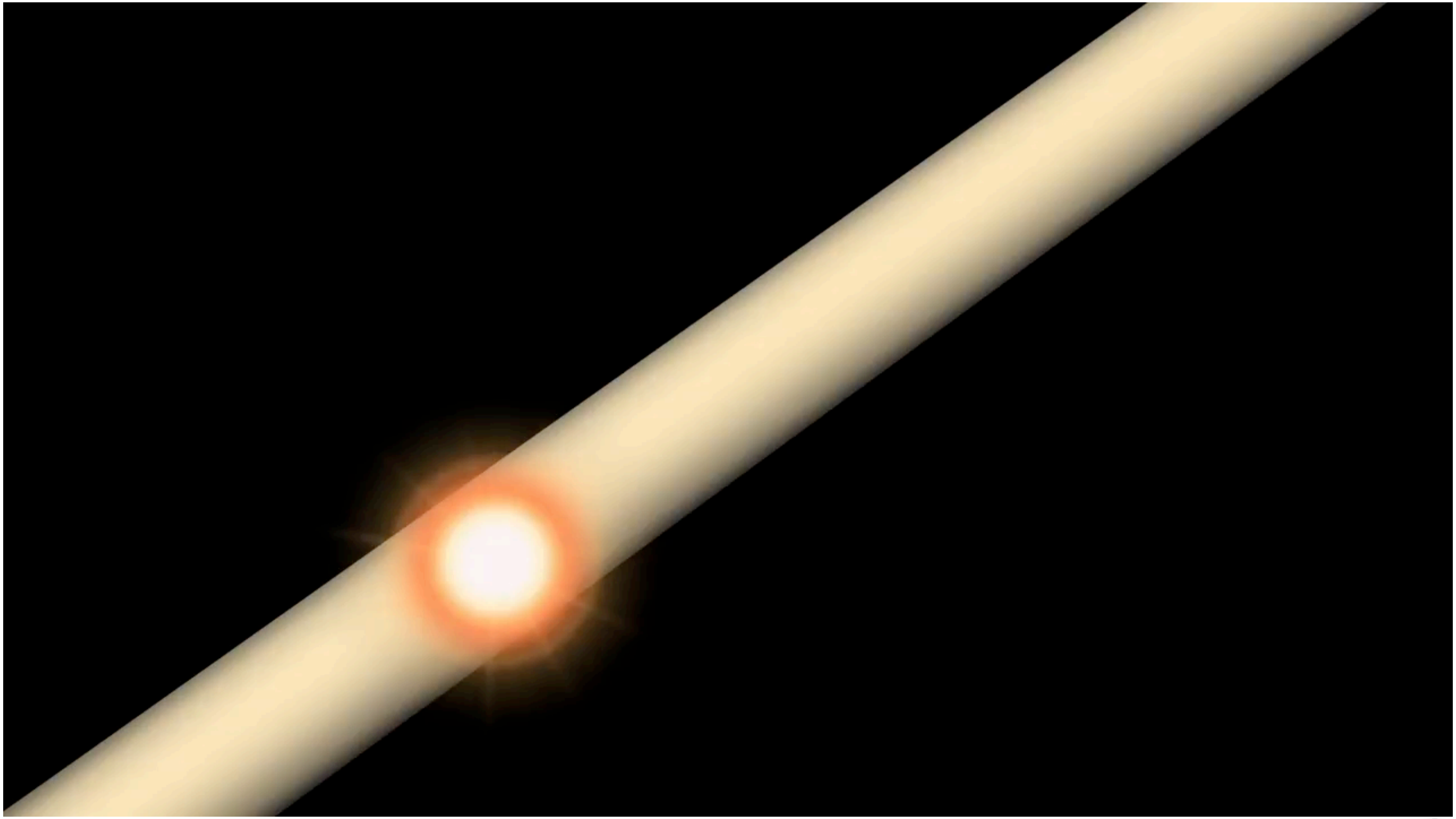




$\sqrt{s}_{NN}$ center-of-mass energy per nucleon pair (NN)

$$s_{NN} = (p_N^{(A)} + p_N^{(B)})^2 = 4m_N^2\gamma^2 \qquad m_N \approx 1\text{GeV} \qquad \gamma_{RHIC} \approx 100 \qquad \gamma_{LHC} \approx 2500$$

Heavy-Ion Collisions



ATLAS Experiment © 2014 CERN

Heavy-Ion Collisions

Very limited control over detailed kinematics of relativistic heavy-ion collisions, only experimental control parameters are

- colliding species e.g. Au,Pb, Zr, Ru
- center of mass energy $\sqrt{s}_{NN}$

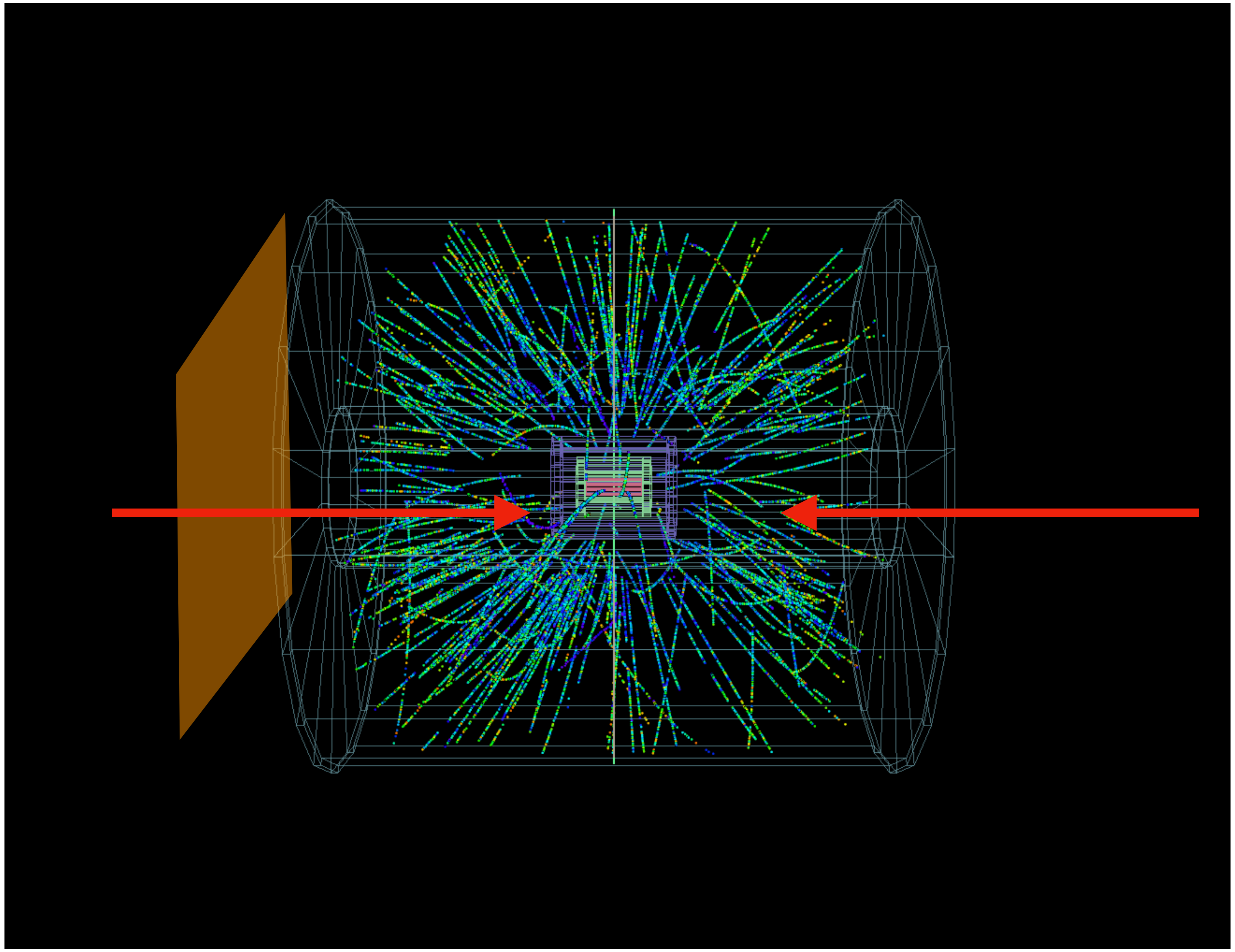
Can observe energy (E) and momenta (p) of

- strongly interacting particles (π,K,p,n,…)
- electro-magnetically interacting particles (γ,e^-,μ^-,..)

produced by the end the collision (no time resolved measurements) within limited kinematic coverage of the detector

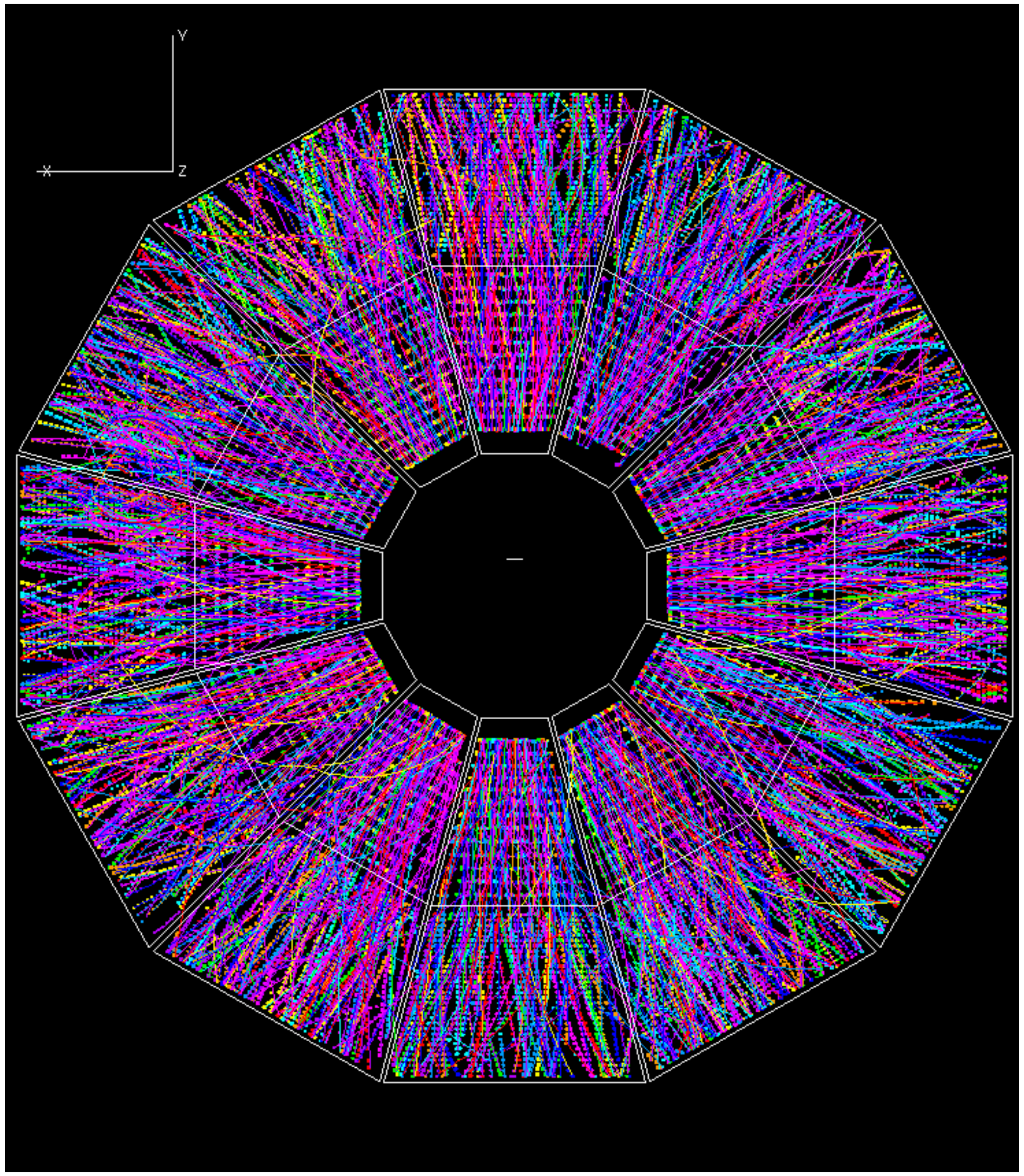
Need sophisticated analysis techniques and detailed theoretical understanding of reaction dynamics to infer information about underlying QCD dynamics

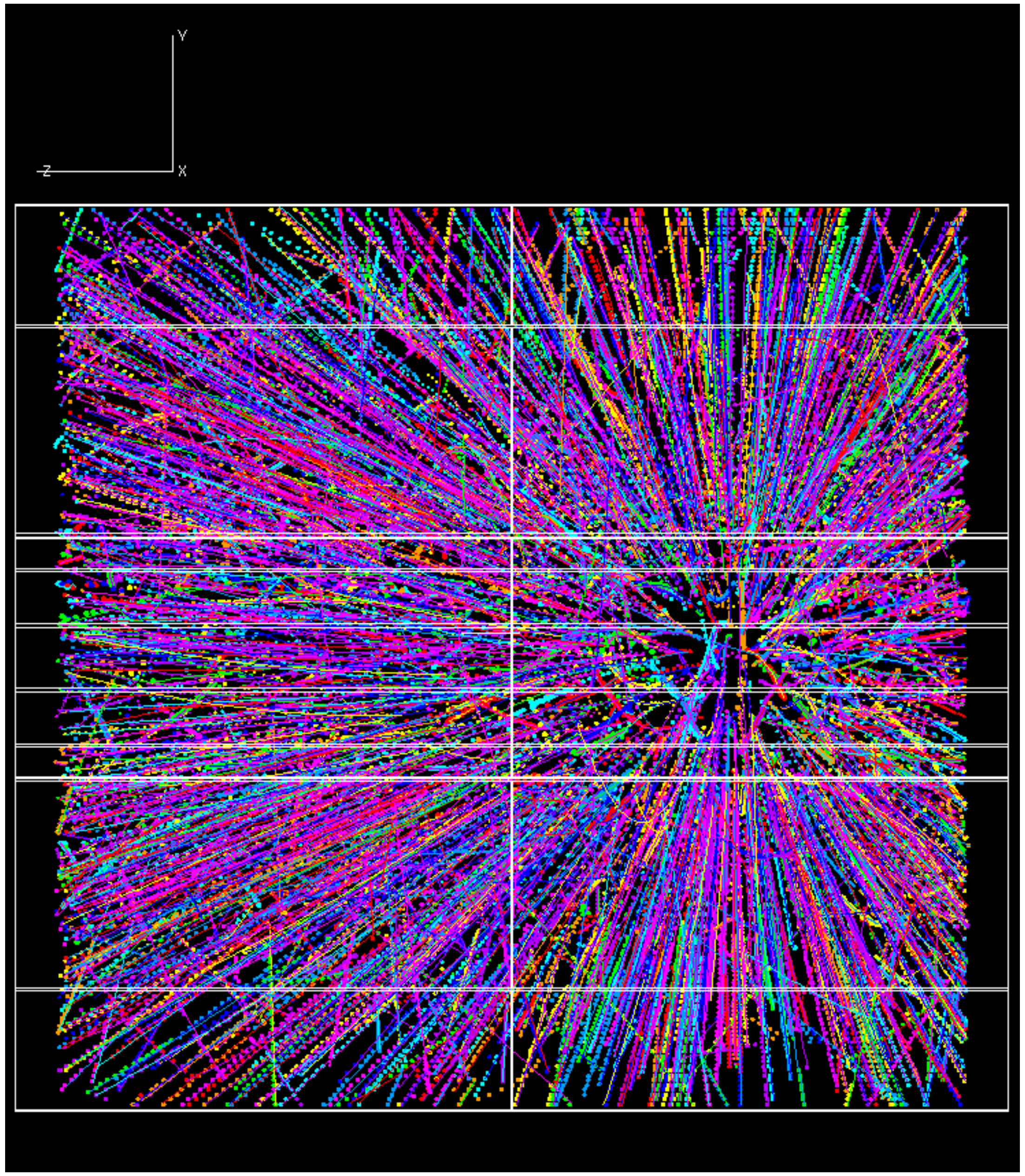
Kinematics



Beam axis defines **longitudinal direction** and plane perpendicular to is referred to as **transverse plane**

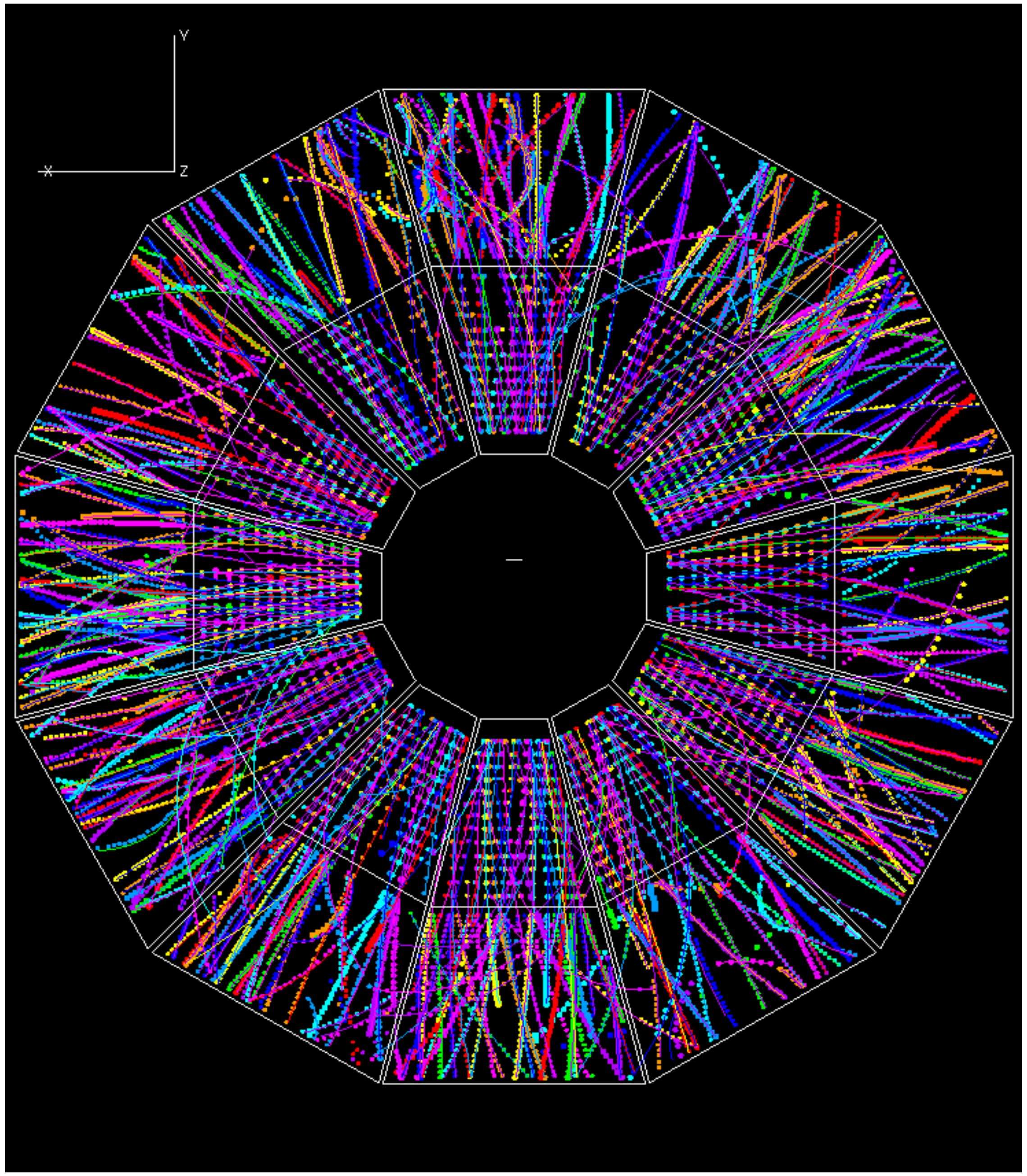
Heavy-Ion Collisions

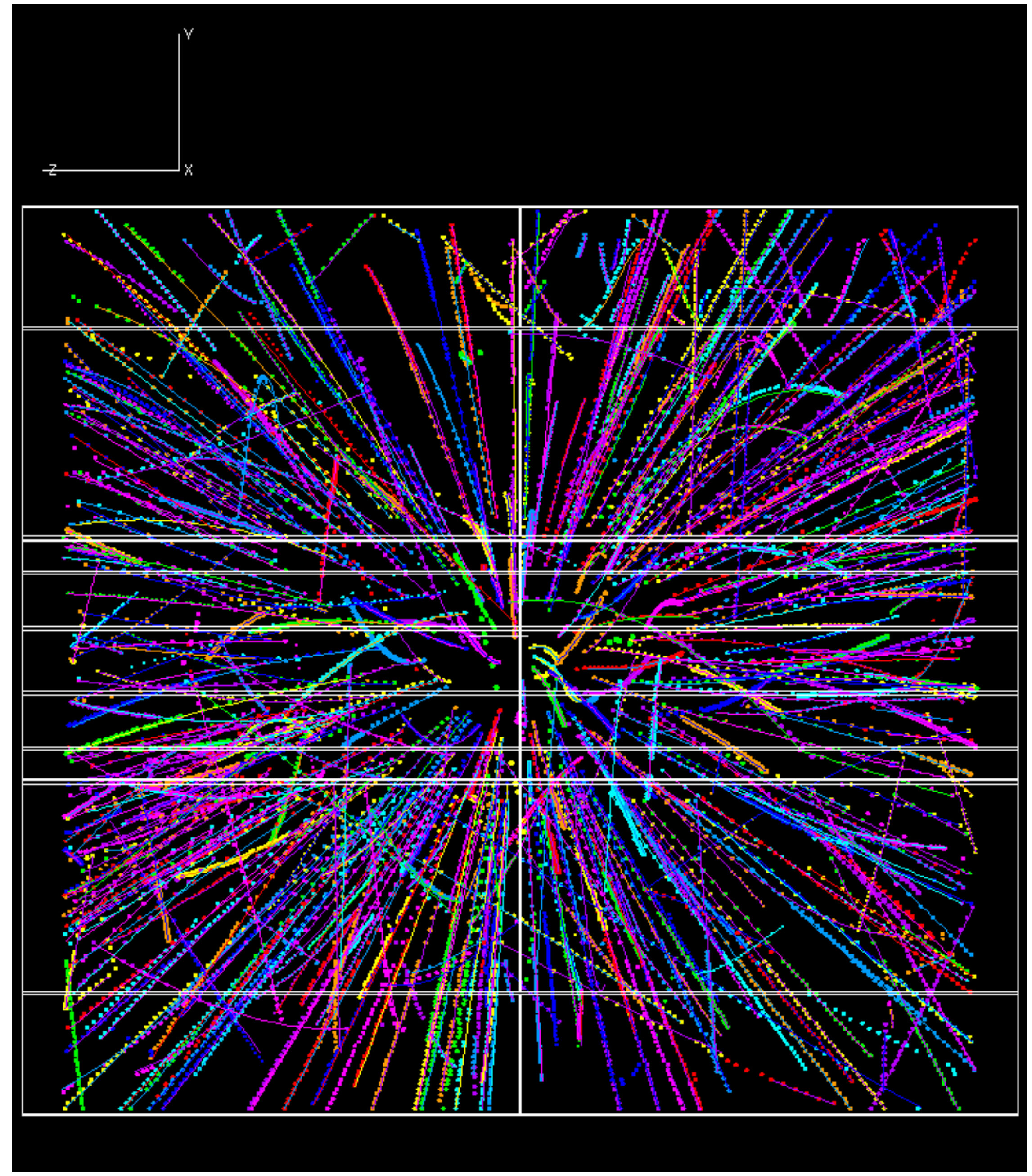




Event display for Au-Au event at $\sqrt{s_{NN}}$ = 130 GeV by the STAR Coll.

Heavy-Ion Collisions



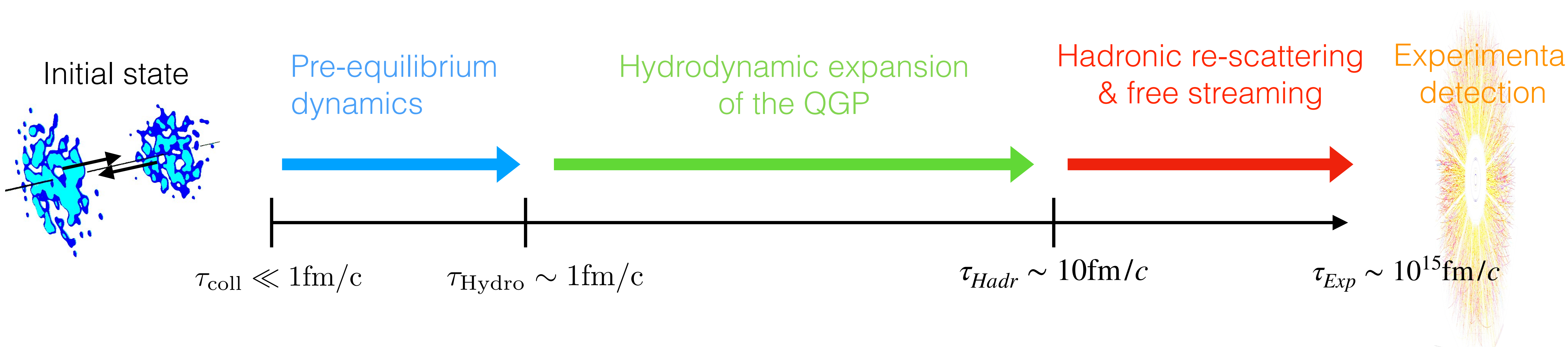


Event display for Au-Au event at $\sqrt{s_{NN}}$ = 130 GeV by the STAR Coll.

Space-time evolution of Heavy-Ion collisions

Dynamical description of Heavy-Ion collisions from underlying theory of QCD remains an outstanding challenge

Standard model of heavy-ion collisions is based on **multi-stage evolution models** that combine different **effective descriptions of QCD** and exploit separation of time scales in reaction dynamics



Hydrodynamics

Hydrodynamics is a universal low energy effective theory
for the macroscopic evolution of many-body systems

Based on conservation laws for energy and momentum (+ charges)

$$\partial_\mu T^{\mu\nu} = 0$$

supplemented by hydrodynamic constitutive relations
that are based on an expansion around (local) thermodynamic equilibrium

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}}(T, u^\mu) + \pi^{\mu\nu}(\partial T, \partial u^\mu)$$

Heavy- (and more so light-) Ion collisions push Hydrodynamics to its extremes, as macroscopic description is applied on time and length scales ~1fm/c

Ideal Hydrodynamics

In (local) thermal equilibrium the energy-momentum tensor is given by

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - p(g^{\mu\nu} - u^{\mu}u^{\nu}) \qquad T^{\mu\nu}|_{LRF} = \textbf{diag}(\epsilon, p, p, p)$$

u^{μ} is the local rest-frame of the fluid, i.e. the frame where the fluid is in thermal equilibrium, normalised as $u^{\mu}u_{\mu} = +1$

$p = p(\epsilon)$ is the thermodynamic pressure determined by EoS

conservation laws

$$\partial_{\mu}T^{\mu\nu} = 0$$

constitutive relations

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - p(g^{\mu\nu} - u^{\mu}u^{\nu})$$

4 Equations for 4 fluid variables

$\longrightarrow \quad \epsilon, u^{\mu}$

Hydrodynamics

By performing projection of conservation equations onto components parallel and perpendicular to u^μ one gets

$$u_\nu \partial_\mu T^{\mu\nu} = u_\mu \partial^\mu e + (e+p)\partial_\mu u^\mu = 0 \qquad De + (e+p)\theta = 0$$

$$D = u_\mu \partial^\mu \qquad \theta = \partial_\mu u^\mu$$

describes local change of energy density due to expansion of the fluid

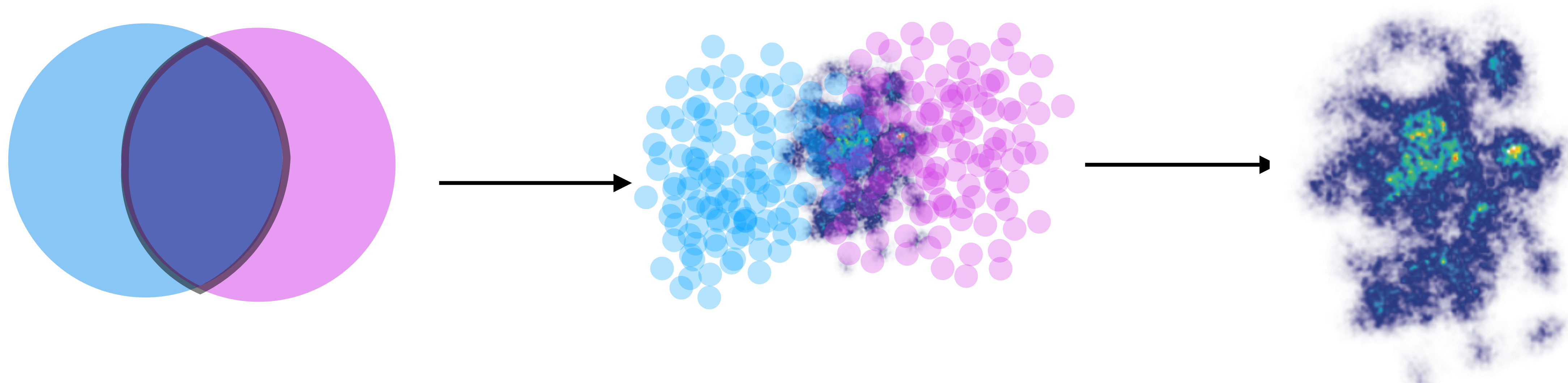
$$\Delta^\alpha_\nu \partial_\mu T^{\mu\nu} = (e+p)Du^\alpha - \nabla^\alpha p = 0$$

describes acceleration of the fluid due to pressure gradients analogous to Newton's law (F=ma)

General picture

During the initial collision a large amount of energy is deposited into the interaction region, which will then cool by expansion into empty space.

Detailed properties of this expansion depend on the properties of the initial state, i.e. **initial conditions for hydrodynamic equations**

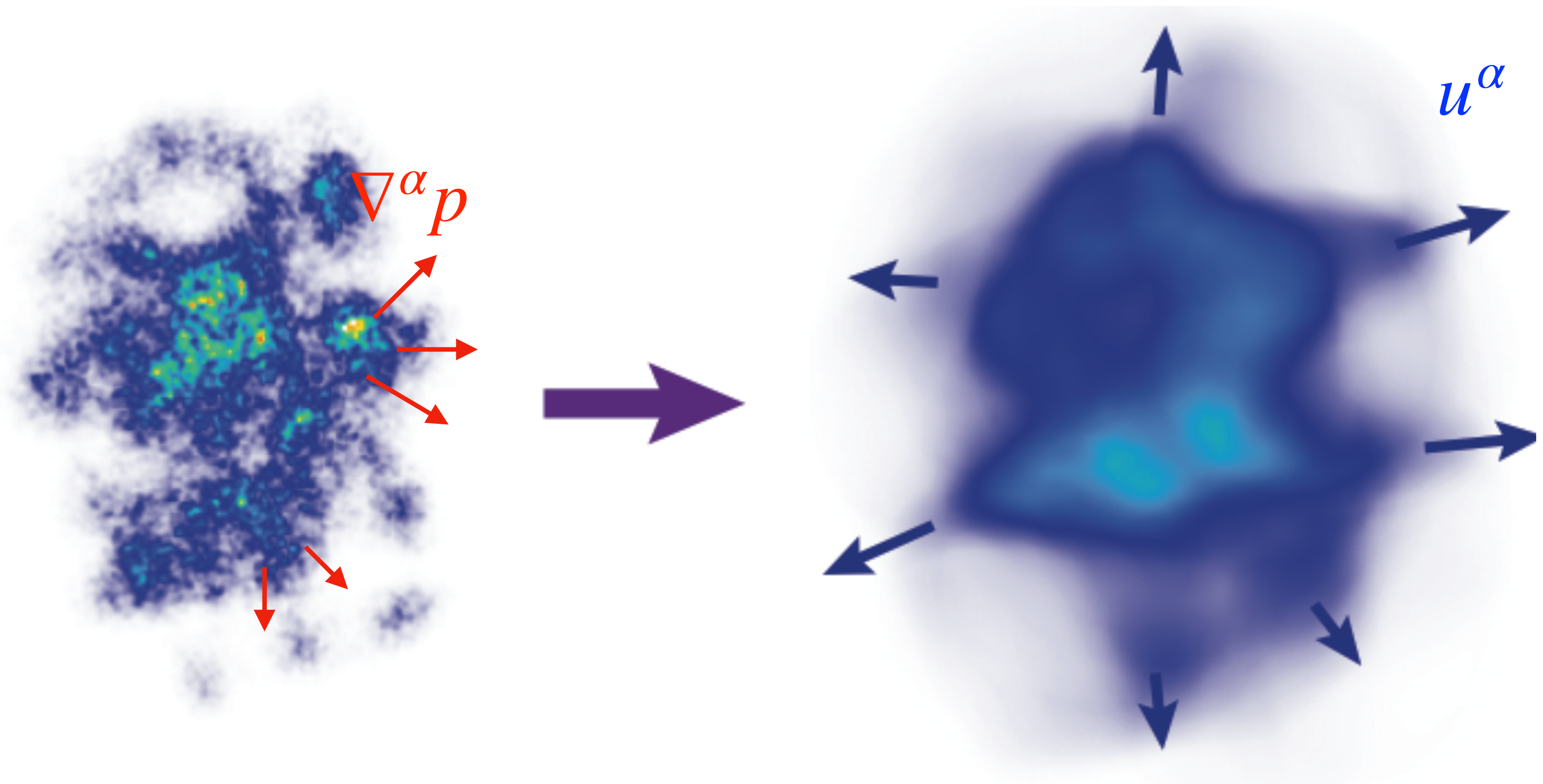


Generally determined by overlapping profiles of colliding nuclei, where off-central collisions create an asymmetric area where energy is deposited

Since nuclei consist of nucleon who's positions fluctuate on an event-by-event basis, there are also (large) **event-by-event fluctuations of the geometry**

Collective flow

Gradients of the pressure drive the transverse expansion of the QGP



$$(e + p)Du^\alpha - \nabla^\alpha p = 0$$

generates an anisotropic collective motion of the QGP fluid

Experimental detection & kinematics

Experiments measure Energy (E) and momenta (p) of produced particles and parametrize momenta as

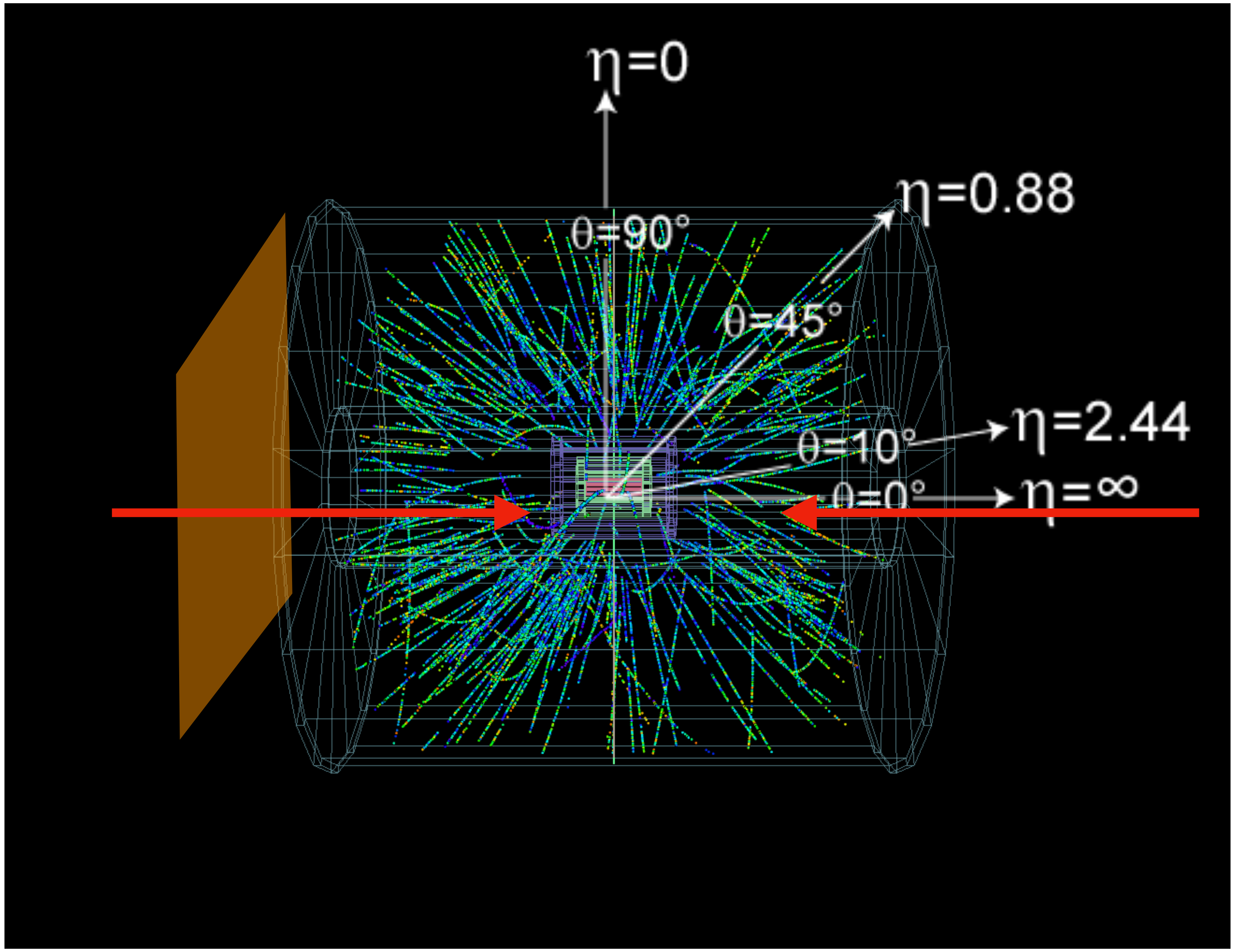
$$p^\mu = (m_T \cosh(y), \mathbf{p}_T, m_T \sinh(y))$$

transverse mass

$$m_T = \sqrt{m^2 + \mathbf{p}_T^2}$$

azimuthal angle Φ

$$\mathbf{p}_T = |\mathbf{p}_T| (\cos(\phi), \sin(\phi))$$



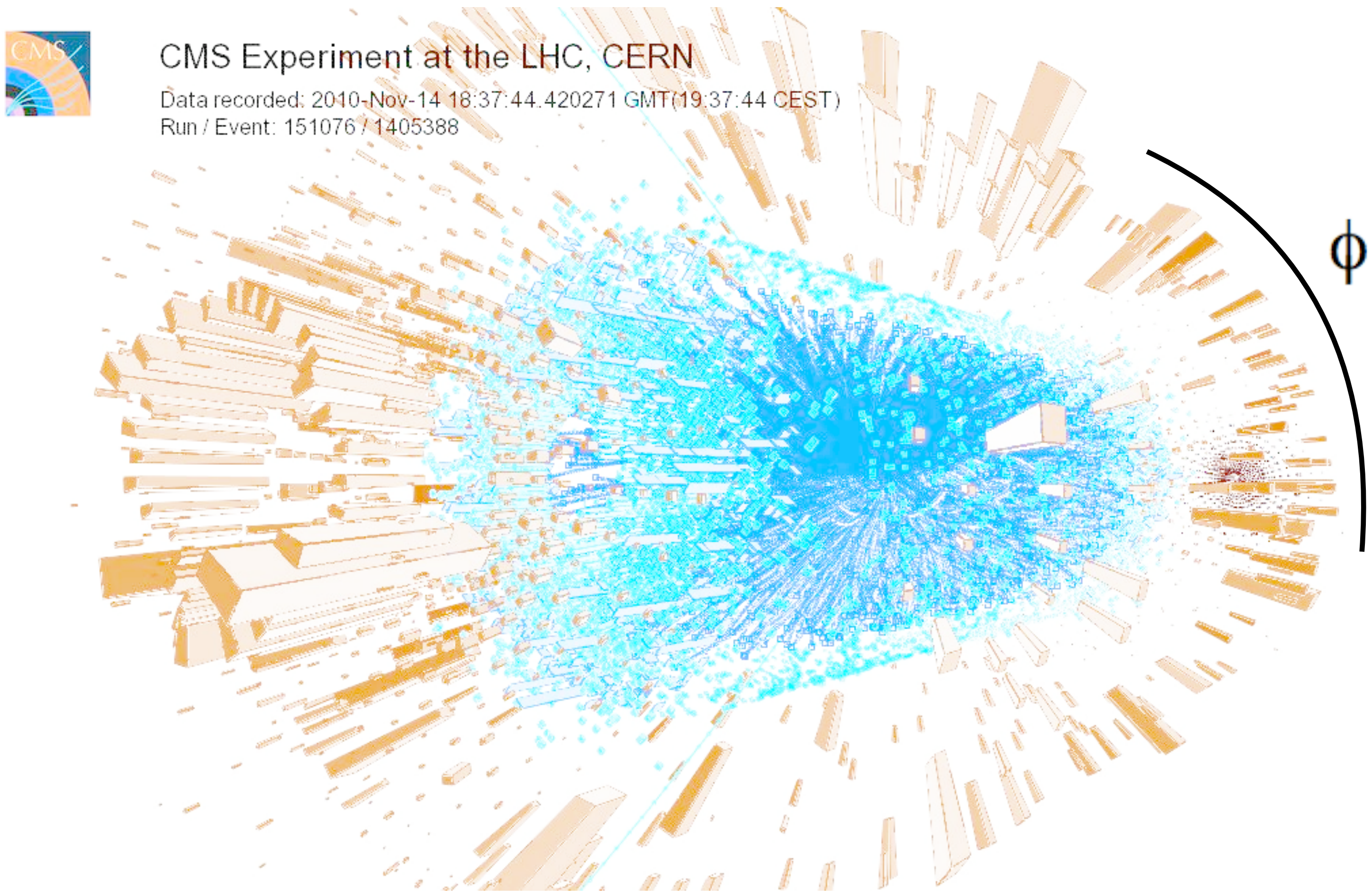
longitudinal rapidity (y) and/or pseudo rapidity (η)

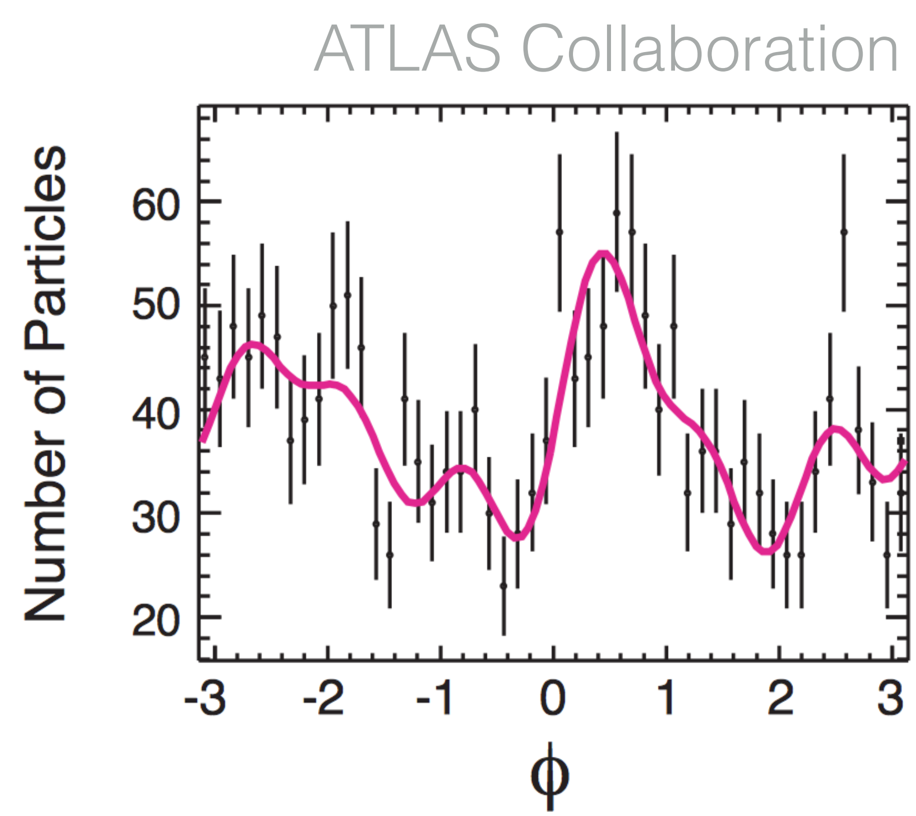
$$y = \frac{1}{2} \log \frac{|E_p| + p_z}{|E_p| - p_z}$$

$$\eta = \frac{1}{2} \log \frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} = \text{arctanh}[\cos(\theta)]$$

Collective flow

Event-by-event spectra of produced particles show pronounced (~10%) modulations in the azimuthal angle Φ in the reaction plane





can be quantified in terms of Fourier coefficients **v_n**

$$\frac{dN}{dy d^2\mathbf{p}_T} = \frac{1}{2\pi} \frac{dN}{dy |\mathbf{p}_T| d\mathbf{p}_T} \sum_{n=0}^{\infty} v_n(|\mathbf{p}_T|) \cos\left(\phi_p - \Psi_n(|\mathbf{p}_T|)\right)$$

Event-plane angle Ψ_n gives preferred direction, weakly dependent on p_T

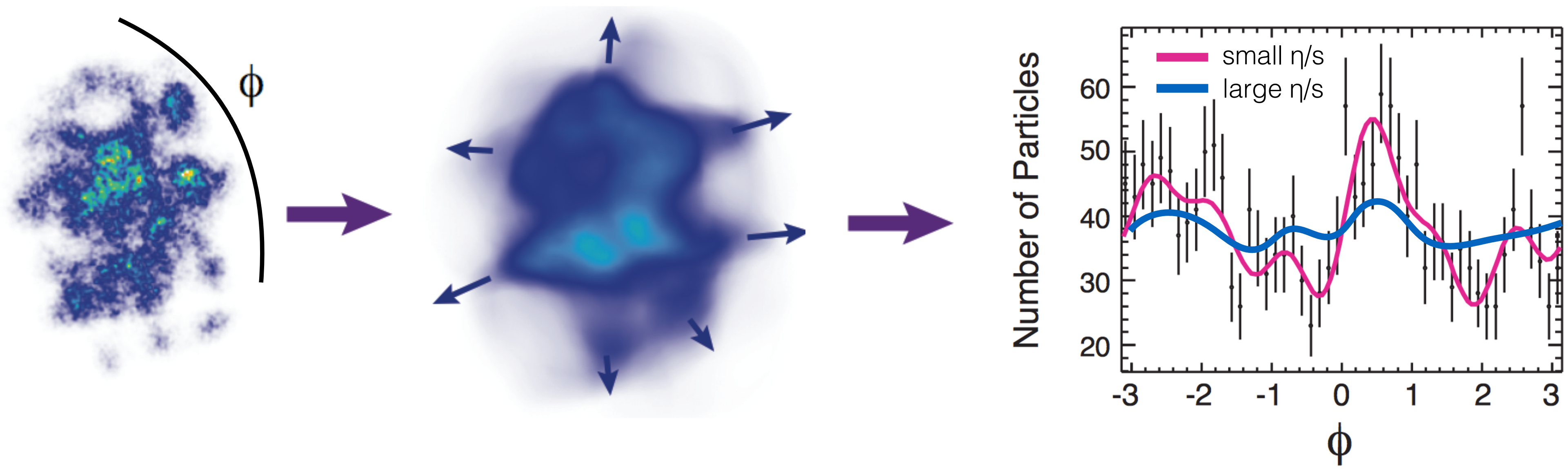
Hydrodynamics

Quantitatively, it is well described by fluid dynamics with a very small viscosity, s.t. the modulation in the initial state yield observed anisotropies in the final state particle distributions

Initial state

Hydrodynamic expansion

Azimuthal anisotropy



B.Schenke, R.Venugopalan, PRL 113 (2014) 102301

Despite phenomenological success, it is still extremely important to understand applicability of hydrodynamics on such microscopic time and distance scales from theoretical point of view