1 Pressure anisotropy in conformal Bjorken flow

Consider a conformal system ($e = 3P \propto T^4$ and $sT = e + P$) in Bjorken flow that follows the first order Müller-Israel-Stewart-type hydrodynamic equations

$$\frac{\mathrm{d}e}{\mathrm{d}\tau} = -\frac{e + P_L}{\tau} , \quad \tau_\pi \frac{\mathrm{d}\pi^\eta_\eta}{\mathrm{d}\tau} = -\pi^\eta_\eta + \frac{4\eta}{3\tau} ,$$

with the longitudinal pressure $P_L = P - \pi^\eta_\eta$, the shear viscosity $\eta = C_\eta s$ and the shear relaxation time $\tau_\pi = C_{\tau\pi} T^{-1}$, where C_η and $C_{\tau\pi}$ are constants. We also introduce the scaled time $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$. Note that we have to assume that $\mathrm{d}\tilde{w}/\mathrm{d}\tau > 0$ at all times for this to make sense.

i) Show that a defined through $\tau\ \mathrm{d}/\mathrm{d}\tau = a(\tilde{w})\tilde{w}\ \mathrm{d}/\mathrm{d}\tilde{w}$ satisfies

$$a = \frac{2}{3} - \frac{f_\pi}{4},$$

where $f_\pi = -\pi^\eta_\eta / e$.

ii) Show that f_π obeys the following evolution equation in terms of $\tilde{w}$.

$$\tilde{w}\left(\frac{2}{3} - \frac{f_\pi}{4}\right)\frac{\mathrm{d}f_\pi}{\mathrm{d}\tilde{w}} = -\frac{4\pi C_\eta}{C_{\tau\pi}}\tilde{w} f_\pi - \frac{16 C_\eta}{9 C_{\tau\pi}} + \frac{4}{3} f_\pi + f_\pi^2$$

Thus, in terms of $\tilde{w}$, the evolution equation for f_π decouples from the energy density.

2 Asymptotic behaviour

Use the evolution equation for $f_\pi(\tilde{w})$ to find

i) the late time asymptotics for $f_\pi(\tilde{w})$, i.e. expand f_π up to linear order in $1/\tilde{w}$.

ii) possible values for f_π at $\tilde{w} \to 0$ and their stability at small finite $\tilde{w}$ by using them to reformulate the evolution equation.

This exercise sheet deals with some of the results of *Phys.Rev.Lett.* 115, 072501 and *Phys.Lett.B* 820, 136478.