1 Dynamical attractor in generalized Bjorken flow

Consider a system of massless particles ($p_0 = E_p = p$) described by the conformal kinetic equation

$$\frac{\partial f}{\partial t} = \frac{\dot{a}}{a} p_z \frac{\partial f}{\partial p_z} - \frac{1}{\tau_R}[f - f_{\text{eq}}(p/T[f])] \, ,$$

that is subject to a weak ($\alpha \ll 1$) harmonic perturbation of the metric

$$a(t) = 1 + \alpha e^{-i\omega t} \, .$$

The system is close to equilibrium and thrown out of equilibrium via the perturbation. We consider corrections to equilibrium quantities X to linear order in α and write them as $\delta X_\omega e^{-i\omega t} \sim \mathcal{O}(\alpha)$ in anticipation of all corrections coming with a phase factor $e^{-i\omega t}$. For example, we will express the phase space distribution as

$$f(t,p) = f_{\text{eq}}(p/T) + \delta f_\omega(p) e^{-i\omega t} \, .$$

i) Show that at linear order in α the solution to the kinetic equation can be expressed as

$$\delta f_\omega = \left[\frac{-i\omega\tau_R \alpha \frac{p_z^2}{p} - \frac{\delta T_\omega}{T} p}{1 - i\omega\tau_R} \right] \frac{\partial f_{\text{eq}}}{\partial p} \, .$$

Hint: Keep in mind that in the evolution equation, f_{eq} is a function of f and will receive corrections due to how δf_ω changes the temperature T of the system.

ii) Calculate the change of the energy density δT^{00}_ω and use the identity $\delta T_\omega / T = \delta T^{00} / 4T^{00}$ to self-consistently determine the induced variation of the temperature $\delta T_\omega / T = -\alpha/3$.
Hint: The energy-momentum tensor is given as

$$T^{\mu\nu} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 p_0} p^\mu p^\nu f$$

and for our system in equilibrium, due to isotropy, one will find

$$e = T^{00} = \frac{1}{2\pi^2} \int_0^\infty \mathrm{d}p \; p^3 f_{\text{eq}} \, .$$

It is a good idea to work in spherical coordinates.

iii) Show that the induced variation of the longitudinal pressure is given as

$$\delta T^{zz}_\omega = \alpha \frac{\frac{4}{5} i\omega\tau_R - \frac{4}{9}}{1 - i\omega\tau_R} T^{00} \, .$$

iv) Find the variation in the shear $\delta\pi^{zz}_\omega$ (Hint: $\pi^{zz} = T^{zz} - p$ and $\delta p_\omega = \delta T^{00}/3$). Use this to find the Green's function

$$G(\omega) = \frac{\partial \delta\pi^{zz}_\omega}{\partial \alpha}$$

and then use the results $T\tau_R = 5\eta/s$ and $\tau_\pi = \tau_R$ for the transport coefficients to compare your result for $G(\omega)$ to the solution in MIS hydro that was discussed in the lecture.

This exercise sheet deals with some of the results of *Eur.Phys.J.C* 76 (2016) 6, 352